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MSC2MatC203x **Seat No :________**

M.Sc.(Maths.) Semester - 2 (*CBCS*) Examination
April/May-2023 (NEW COURSE)
Topology -2 (Core)

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Q.1(A) Answer the following questions. (Any Two) (10)

1. Let X be a T_1 space, and let A be a subset of X. Then show that $x \in X$ is a limit point of A if and only if every neighbourhood of x contains infinitely many points of A.
2. Show that X is Hausdorff space if and only if the diagonal $\Delta = \{x \times x : x \in X\}$ is closed in $X \times X$.
3. Show that the T_1 axiom is equivalent to the condition that for each pair of points of X, each has a neighbourhood not containing the other.

Q.1(B) Answer the following question. (Any One) (04)

1. Show that every finite point set is closed in Hausdorff space
2. Show that the product of two Hausdorff spaces is Hausdorff.

Q.2(A) Answer the following questions. (Any Two) (10)

1. Show that X is regular if and only if for given a point x of X and a neighbourhood U of x, there is a neighbourhood V of x such that $\bar{V} \subset U$.
2. Show that a regular space with countable basis is normal.
3. Show that every closed subspace of normal space is normal.

Q.2(B) Answer the following question. (Any One) (04)

1. Show that a subspace of regular space is regular.
2. Show that every compact Hausdorff space is Normal.

Q.3(A) Answer the following questions. (Any Two) (10)

1. Show that the product of two compact spaces is compact.
2. Let A be an open cover of the metric space (X, d). Show that if X is compact, then there is $\delta > 0$ such that for each subset of X having diameter less than δ is contained in an element of A.
3. Show that every compact space is limit point compact.

Q.3(B) Answer the following question. (Any One) (04)

1. Show that every closed subspace of compact space is compact.
2. Show that the continuous image of compact space is compact.

Q.4(A) Answer the following questions. (Any Two) **(10)**

1. Let $f: X \to Y$ be continuous map from compact metric space (X, d_X) to metric space (Y, d_Y). Then show that f is uniform continuous.
2. Let X be a metrizable space. If X is limit point compact, then show that X is sequentially compact.
3. Let X be a metrizable space. If X is sequentially compact, then show that X is compact.

Q.4(B) Answer the following question. (Any One) **(04)**

1. Let (X, d) be a metric space and $A \subset X$ be a nonempty, then show that $d(x, A) = 0$ if and only if $x \in \bar{A}$.
2. Let (X, d) be a metric space, let K be a compact subset of X, and let A be a subset of X. Then show that there is $k \in K$ such that $d(k, A) = \inf\{d(x, y) : x \in K, y \in A\}$.

Q.5(A) Answer the following questions. (Any Two) **(10)**

1. Show that a metric space (X, d) is complete, if every Cauchy sequence has convergent subsequence.
2. Let X be a topological space, and let (Y, d) be a complete metric space. Then show that the set $\mathfrak{B}(X, Y)$ of all bounded functions from X to Y is a complete metric space under the uniform metric.
3. Let (X, d) be a metric space. Then show that there exists an isometric imbedding of X into a complete metric space.

Q.5(B) Answer the following question. (Any One) **(04)**

1. Show that the closed metric subspace of a complete metric space is complete.
2. Show that every compact metric space is complete.

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